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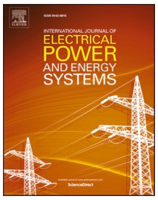

Peer-to-peer market with network constraints, user preferences and network charges

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ABSTRACT

With an increase of distributed generation growing attention is paid to the possibilities of its utilization in the network. The peer-to-peer market represents one of the possible ways to address this question. Largely driven by distributed ledger technologies, the peer-to-peer market architectures ignored network constraints for a long time, paying more attention to the organization of the financial transactions. In this paper we propose a peer-to-peer market design, incorporating network constraints, user preferences, and trade-independent network fees. In this way, we ensure a meeting of three requirements critical to the practical implementation of the peer-to-peer market as secure operation, consumer-centric nature of the market, and the provision of benefits for the grid. We develop a distributed procedure and demonstrate the applicability of the proposed algorithm using the IEEE 39-bus power system, and compare it with the correction-based algorithm.

1. Introduction

Today power systems operate in more stressed conditions than was forecasted at the planning stage. With the growth of energy consumption the generation reserves and transmission capacities did not always increase appropriately. Failure to conduct necessary long-term planning studies and insufficient resources that can be relied upon to meet demand in certain hours are some of the reasons for blackouts [1,9]. To introduce additional generation capacities to the system, Market Operators consider possibilities for deeper integration of distributed energy resources (DER). DER represent one of the new technologies attractive for users and, if correctly integrated, capable of releasing the network stress and minimizing the number and impact of blackouts [1].

To uncover the potential of distributed generation, novel market structures besides a feed-in-tariff operation could be proposed. Authors in [27] identify three possible market models to integrate prosumers and distributed generators: peer-to-peer (P2P) market, prosumer-to-grid operation, and utilization of organized prosumer groups. Each of these approaches has its specifics [27]. This work focuses on the design of the P2P electricity market, offering more independence and freedom of action to market participants. The P2P trading scheme enables new types of services and proposes additional value as differentiated contracts, enforced consumer preferences, and increased utilization of distributed generation [17].

The organization of the P2P trading scheme is recently emerged and still an open question. What are the requirements for the P2P electricity market? Authors in [32,29] emphasize several important aspects of P2P market organization necessary for its practical implementation. They suggest a market to be consumer-centric at the same time providing benefits for the grid. Special attention is paid to the security requirements, possible privacy issues, complications related to asynchronous communication in the system, and computational complexity of the proposed solutions. Applicable model of the P2P market has to be able to work with incomplete information and need to incorporate the physical laws. As summarized in [31], different techniques can be applied to design the P2P trading schemes as auction based, game theory based and constrained optimization approaches.

While the requirements for the P2P market have been formulated, the development of the P2P market concept to meet these requirements is in progress. In a considerable measure, interest in P2P markets was stimulated by the emergence and spread of distributed ledger technologies, particularly, blockchain technology, and the investigation of its possible applications in the energy sector [11,26,33]. This explains that the early works in this direction focus on the organization of the financial transactions or other aspects of the P2P market [2,18], ignoring network constraints. In the meantime, the need to distinguish the market-clearing dispatch and feasible dispatch is critical for secure market and network operation. Additionally, for the P2P market it is

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Table 1
Taxonomy table of scientific articles addressing the issues of the P2P market.

Source	Approach	Congestion management	Preferences	Benefits for the grid	Distributed procedure
[25]	Real-time and forward markets based on the distributed price-adjustment process	–	utility-maximising	–	+
[34,28]	Coordinated multilateral trading model. The reliability is ensured by the SO	permission-based	–	–	+ (–)
[6]	Optimization-based P2P market with exogenous cost allocation	exogenous, limited application	–	+	+
[16]	Continuous double auction mechanism	permission-based	–	–	+ (–)
[24]	Distributed price-directed optimization procedure, using ADMM	–	introduced energy classes	–	+
Current work	Optimization-based P2P market with built-in congestion management	built-in	+	+	+

important to avoid, as far as it's possible, market interventions. It is essential to eliminate the revealing of information, which may lead to welfare losses and gaming, mainly the cost functions of agents.

The second issue relates to the point that bilateral models in the original form do not include payments for grid maintenance and modernization. However, this aspect and an ability to reach energy policy objectives, usually through tariffs, play an essential role in practical implementation. For this purpose, when energy consumers pay directly to the generators, the trade-independent network fees should be envisioned and introduced.

Thirdly, the P2P market architecture needs to be adapted to incorporate agents' preferences. User preference in this context is the willingness to trade with the specific agents more than others. An additional instrument should be proposed to integrate the decision rules based on the type of generation, CO_2 footprint, distance, time preferences, and others.

In this article we propose a P2P market design that addresses the three issues above. It works as a unified framework incorporating network constraints, user preferences, and trade-independent fees responsible for network charges, energy policies, and taxes. Some of these aspects are discussed in the literature. We present the main works in Table 1. These articles differ in the applied technique to the organization of the P2P market (continuous double auction mechanism, distributed price-adjustment process, optimization-based market, etc.) and in coverage of the three previously mentioned issues. The + (–) sign in Table 1 means that the proposed decentralized algorithm is supplemented by the centralized permission procedure.

As stated early, the physical layer is pivotal in power markets, and adherence to operational limits and network constraints is obligatory. Existing approaches to address this question can be divided into three groups. Firstly, one can use an iterative procedure when the market operator verifies market participants' trades to enforce line limits like in [34,28]. A similar permission-based structure is applied in [16]. Such approaches require the presence of a third-party entity to validate the transactions.

Another logic accounts for network constraints in an endogenous manner, including them in the constraints of the optimization problem, similarly to the optimal power flow task. In this case, information about bus voltage angles across the network is required to set the trades. This implies a tight connection with the system operator (SO) side, resulting in weakening the peer-to-peer market concept. Besides that, special attention has to be paid to the decentralization of the algorithm.

The third approach [6] enforces network constraints in an exogenous manner, supplementing objective function with the trade-dependent network charge component, which can be used to release the stress on the grid. The methodology, proposed in [6], is optimization-based with the power balance ensured through reciprocity constraints. However, introducing fees does not guarantee an absence of congestion [6]. In this way, exogenous congestion management does not resolve the problem of line overloads. Additionally, it negatively impacts the total amount of trades, and social welfare [6]. At the same time, we consider the framework itself as a great inspiration and the starting point for future analysis. Besides that, we observe that it is possible to utilize the form of the term used in [6] for congestion management to introduce user preferences. The constrained optimization framework, based on the mathematical programming techniques, in a certain sense, can be considered as a benchmark for other methods, indicating the maximum of the total welfare.

In this article we propose an optimization-based peer-to-peer market design that solves the three problems mentioned early. To ensure the feasibility of market outcome and the adherence of power flow limits, we apply the matrix of loading vectors (power transfer distribution factors (PTDF) approach) in a built-in form and exploit an exogenous approach to include users' preferences and network charges. We for the first time develop a distributed procedure for the P2P market with built-in congestion management and demonstrate the applicability of the proposed approach using the IEEE 39-bus power system, and compare it with the correction-based algorithms. In this way, we provide a combination of the algorithm's characteristics that make it attractive to practical implementation, such as secure operation, consumer-centric nature of the market, and the provision of benefits for the grid.

Summing up, in this article we

- Solved the problem of congestion management for the optimization-based P2P market applying the matrix of loading vectors in an endogenous manner and avoiding intermediate power flow calculations and the permission procedure;
- Effectively introduced user preferences in a way suitable for different types of preferences;
- Developed the original distributed procedure for the proposed algorithm. It shows the applicability of the built-in methodology for energy systems;
- Compared the suggested approach with the correction-based algorithms and provided insights about possible challenges for correction-based options giving hints for the industry;
- Incorporated into the model trade-independent network fees to provide benefits for the grid and ensure money collection for grid maintenance and modernization.

The rest of the paper is organized as follows. In Section 2 we introduce notations and state initial problem formulation. Section 3 proposes the P2P electricity market architecture with network constraints, user preferences, and network charges. In Section 4 we describe the distributed P2P market algorithm. Section 5 contains information about the test case and presents the simulation results. In Section 6 we discuss additional aspects of the proposed methodology. Section 7 contains conclusions of the work.

2. Design of P2P energy market

We consider a P2P market as a community of rational agents with flexible generation and consumption. The model can be extended to include "must consume" generation, and "must supply" loads. The proposed formulation is compatible with the presence of prosumers inside the community.

2.1. Notations and network model

The network consists of a set of buses $B = \{1, \dots, N_b\}$ and a set of lines $L = \{1, \dots, N_l\}$, where N_b and N_l are the number of buses and lines, respectively. Generators and flexible loads in the system form a set of agents of the P2P market $\Omega = \{1, \dots, N_\Omega\}$ for which $\Omega = \Omega_g \cup \Omega_c$, where Ω_g is a set of generators, Ω_c is a set of consumers, N_Ω denotes the number of agents participating in the market. Some agents can be located on the same bus; at the same time, some buses may not have agents. To describe the correspondence between buses and agents we introduce an incidence matrix $\mathbf{I} \in \mathbb{R}^{N_\Omega \times N_b}$, for which

$$\mathbf{g} = \mathbf{I}^\top \mathbf{p} \tag{1}$$

where $\mathbf{g}$ is a vector of bus power injections, $\mathbf{p}$ is a vector of the total amount of power traded by agents. Matrix $\mathbf{I}$ is constructed in a such way that $\mathbf{I}_{ij} = 1$, if an agent i is connected to the bus j, otherwise $\mathbf{I}_{ij} = 0$.

2.2. Problem formulation

P2P market as a set of multiple bilateral trades between the agents could be described in the form of an optimization problem [6] with the goal to minimize the total cost equal to the sum of individual costs $f_n(p_n)$ of agents within community Ω (2a)

$$\min_{\mathbf{P}, p_{n\in\Omega}, \theta_{i\in B}} \quad \sum_{n\in\Omega} f_n\left(p_n\right) \tag{2a}$$

$$\text{s.t.} \quad \mathbf{P} = -\mathbf{P}^\top, \tag{2b}$$

$$p_n = \sum_{m\in\omega_n} p_{nm}, \ \forall n \in \Omega, \tag{2c}$$

$$\underline{p_n} \le p_n \le \overline{p_n}, \ \forall n \in \Omega, \tag{2d}$$

$$p_{nm} \geqslant 0, \ \forall n \in \Omega_g, \tag{2e}$$

$$p_{nm} \leqslant 0, \ \forall n \in \Omega_c, \tag{2f}$$

$$l_{ij} = B_{ij}(\theta_i - \theta_j), \ (i,j) \in L, \tag{2g}$$

$$\underline{l_{ij}} \leqslant l_{ij} \leqslant \overline{l_{ij}}, \ \left(i,j\right) \in L, \tag{2h}$$

where p_n represents a total amount of power traded by agent n within the operational limits (2d). It is a result of the summation of all trades p_{nm} within the trading partnership set ω_n of agent n, $\omega_n \subseteq \Omega$. Matrix $\mathbf{P}$ collects all possible bilateral trades within the P2P community with the zero-valued elements p_{nm} if the agent n does not trade with agent m

$$\mathbf{P} = \begin{pmatrix} p_{11} & \dots & p_{1N_\Omega} \\ \dots & \dots & \dots \\ p_{N_\Omega 1} & \dots & p_{N_\Omega N_\Omega} \end{pmatrix}.$$

In this way, matrix $\mathbf{P}$ describes a map of trades. Eq. (2b) ensures power balance of each trade in the system. We assume that p_{nm} is positive when an agent n is selling electricity (2e), and negative when it is consuming power (2f), also $p_{nn} = 0$. The model can be extended to integrate the prosumers as the agents without the prescribed sign of the total trade. Optimization problem (2) uses DC approximation of power flows (2g), where l_{ij} is a power flow through the line (i,j), θ_i and θ_j are the voltage angles at the ends of the line, B_{ij} is the line parameter. Transmission constraints are enforced by (2h), where $\overline{l_{ij}}$ and $\underline{l_{ij}}$ are the upper and lower power flow limits. Network constraints in the form (2g)–(2h) is a classical representation of operational requirements. However it is hardly suitable for the application in the P2P market. To address this issue, we propose a methodology based on the matrix of loading vectors to guarantee that bilateral trades do not violate network constraints.

In the form (2) P2P market is close to the traditional optimal power flow task. The fundamental difference consists of introducing the matrix $\mathbf{P}$, which makes it possible to potentially individualize prices per trade reflecting the nature of the P2P market. As summarized in [31], different techniques can be applied to design the P2P trading schemes as auction based, game theory based and constrained optimization approaches. The use of a particular technique depends on the subject of the analysis.

The constrained optimization framework based on the mathematical programming techniques in a certain sense can be considered as a benchmark for other methods, indicating the maximum of the total welfare. It provides a solution equivalent to the solution of the set of individual agent's problems and applicable for non-strategic agents, for markets without game-related issues, such as cooperation and market power. We will use a constrained optimization framework to develop a distributed P2P market architecture encompassing network constraints, user preferences, and network charges.

3. P2P electricity market with network constraints, user preferences and network charges

In this paper we develop a distributed optimization framework for the P2P electricity market. We propose the utilization of a matrix of loading vectors to deal with network constraints. We introduce user preferences and exploit a regularization function approach to address the second issue of the P2P market - a collection of trade-independent fees.

3.1. P2P market with network constraints

The P2P electricity market represents a set of bilateral trades delivered by the physical infrastructure. Although the early works focusing on other aspects of the P2P market ignore network constraints, they have to be accounted for secure network and market operation. An optimization problem (2) is a general formulation of the P2P market with endogenous enforcing of line limits. It can be addressed and solved directly. However, in the current form, the calculation of the agents' trades requires information about bus voltage angles across the network at each step of solving the algorithm. Usually this implies a tight connection with DSO, weakening a concept of the P2P market. Besides that, special attention has to be paid to a decentralization of the algorithm. This work suggests a simple suitable for decentralization reformulation of the problem (2) using the matrix of loading vectors for accounting network constraints. This matrix collects the sensitivities of lines' power flows to the changes in bus power injections. This step eliminates the need for θ as an optimization variable. In this way, we establish the direct relationship between the trades and the power flows, eliminating the calculation of the power flows in a pure form, speeding up and simplifying calculations, and enabling the decentralization of the algorithm. Firstly the utilization of loading vectors was described in [34] to determine the feasibility boundaries for profitable trades of brokers. We propose to use this logic to account for network constraints in a built-in manner.

Power flows (2g) in a matrix form can be expressed as

$$\boldsymbol{l} = \mathbf{B}\mathbf{C}^\top \boldsymbol{\theta}, \tag{3}$$

where $\boldsymbol{l}$ denotes the values of power flows, $\mathbf{B}$ represents the diagonal matrix with non-zero elements equal to $B_{ij} = \frac{1}{x_{ij}}$ with line reactances x_{ij}, $(i,j) \in L$. Matrix $\mathbf{C} \in \mathbb{R}^{N_b \times N_l}$ is an incidence matrix with the $C_{i,\varepsilon} = 1$ if line $\varepsilon = (i,j)$ is from bus i to some bus j, $C_{i,\varepsilon} = -1$ if line $\varepsilon = (k,i)$ is from some bus k to bus i, and $C_{i,\varepsilon} = 0$ otherwise.

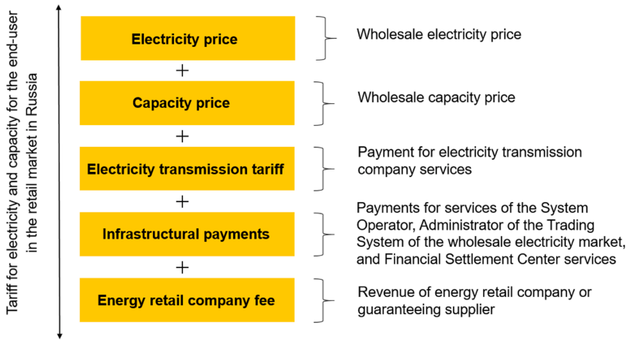


Fig. 1. The structure of tariff for electricity and capacity for the end user in the retail market in Russia [13].

In the same notations, bus power injections **g** can be described as a function of angles

$$\mathbf{g} = \mathbf{C}\mathbf{B}\mathbf{C}^{\top}\boldsymbol{\theta}. \tag{4}$$

Then after selection of the slack bus

$$\boldsymbol{\theta} = \mathbf{S}^{-1}\mathbf{g}, \tag{5}$$

where $\mathbf{S} = \mathbf{C}\mathbf{B}\mathbf{C}^{\top}$. Using this expression in (3), power flows as a function of bus power injections take a form

$$\boldsymbol{l} = \mathbf{B}\mathbf{C}^{\top}\mathbf{S}^{-1}\mathbf{g}, \tag{6}$$

or in other notations $\boldsymbol{l} = \mathbf{A}\mathbf{g}$, where $\mathbf{A} = \mathbf{B}\mathbf{C}^{\top}\mathbf{S}^{-1}$. Matrix $\mathbf{A} \in \mathbb{R}^{N_l \times N_b}$ is a matrix of loading vectors, in which the rows contain sensitivities of the lines' power flows to the changes of bus power injections. Then power flow constraints (2 g)-(2 h) read as

$$\left|\mathbf{A}\mathbf{g}\right| \leqslant \bar{\boldsymbol{l}}. \tag{7}$$

Applying the incidence matrix **I**, the set of inequalities (7) takes a form

$$\left|\mathbf{G}\mathbf{p}\right| \leqslant \bar{\boldsymbol{l}}, \tag{8}$$

where $\mathbf{G} = \mathbf{A}\mathbf{I}^{\top}$, (8) describes the influence of agents' power injections on the line loading.

3.2. P2P market with network charges

The optimization problem (2) with substituted (2g)–(2h) by (8) accounts for network constraints in an endogenous manner. It results in the nodal prices, encompassing both generation related and congestion-related costs. The structure of electricity and capacity tariff for the end-users in Fig. 1 shows that apart from the payments for electricity, it includes several extra fees. Electricity retail price additionally covers the cost of purchasing capacity on the wholesale market, electricity transmission tariff with the share of one third of total tariff, infrastructural payments for operational, market-related and financial supporting services, and revenue of energy retailers. Similarly, the P2P market assumed working in parallel with the existing supply scheme can not avoid several additional charges. Primarily they relate to the network utilization and administration of the P2P electricity trading platform. To ensure that the P2P market is cost recovering for the grid agents' energy payments need to include an extra part responsible for the trade-independent network and policy-related charges, as well as payments for administration of the P2P trading platform. For this purpose we introduce β_n^0 in the objective function of (2). It can be defined for each agent n independently. In general case, β_n^0 is the sum of the individual terms responsible for certain additional payments. The values β_n^0 can also be used as an instrument for the implementation of energy policies. It should be noted that in the current trade-independent form, β_n^0 does not change the decision as the constant term in the objective function. Having this term is useful for individual agents to get information about total payments. In practice, individual agents may set the trading limit in terms of the maximum acceptable amount of total payment. At the same time, there could be proposed the alternative approaches to account for network charges: depending on whether an agent trades at this market-clearing moment or not; the network charges could be trade-dependent, and others. For example, there could be assigned a limit for total trade for each agent indicating when the agent utilizes the grid more than planned - for instance, in the situation with significant wind fluctuations for wind farms. In these cases, network fees could be increased. However, the trade-dependent fees currently are not welcomed in the industry.

3.3. P2P market with user preferences

The P2P market assumes that every agent is free to choose the contractor. The framework (2) supports the individualized prices per trade and, in the current form, implements the cost-based decisions. However, users' preferences considered as the willingness to trade with the specific agents more than others are not limited to this approach. There should be proposed an additional instrument to integrate other decision rules based on the type of generation, CO_2 footprint, zone of trading, time, and others.

In this article we exploit a regularization function approach and assign an additional cost to the undesired trades in the form of $\beta_{nm}p_{nm}$, where β_{nm} represent preference coefficients. The current work explores a preference to trade with the neighboring agents reflecting the tendency to support on-place generation [21] and eliminate unnecessary power transfer. Following this logic preference coefficients β_{nm} are taking proportional to the electrical distance between the agents d_{nm}. Under electrical distance we assume Power Transfer Distance, described in detail in [10,6], and calculated as $d_{il} = \sum_{(q,x)\in L}\left|D_{il,qx}\right|$, where $D_{il,qx}$ is the power transfer distribution factor on branch $q - x$ for injection at node i and withdrawal at node l.

In that case, $\beta_{nm} = \pm\frac{s_{nm}d_{nm}}{2}$ for agents n and m within the P2P community Ω, where s_{nm} represents the distance unit fee for the particular trade. The higher the value s_{nm}, the stronger is the distance preference of agent n. The sign of β_{nm} has to be chosen to satisfy $\beta_{nm}p_{nm} \geqslant 0$. In the general form, one can introduce a matrix of preference coefficients

$$\mathbb{B} = \begin{pmatrix} \beta_{11}^{dist} + \dots + \beta_{11}^{CO_2} & \dots & \beta_{1N_\Omega}^{dist} + \dots + \beta_{1N_\Omega}^{CO_2} \\ \dots & \dots & \dots \\ \beta_{N_\Omega 1}^{dist} + \dots + \beta_{N_\Omega 1}^{CO_2} & \dots & \beta_{N_\Omega N_\Omega}^{dist} + \dots + \beta_{N_\Omega N_\Omega}^{CO_2} \end{pmatrix},$$

where elements of the matrix represent a superposition of the existing agent preferences as distance-based β_{nm}^{dist}, CO_2-based $\beta_{nm}^{CO_2}$ and others.

3.4. P2P Electricity Market with Network Constraints, User Preferences and Network Charges

Applying proposals from previous paragraphs, we can design the P2P market with network constraints, agents' preferences, and trade-independent fees as the following optimization problem

$$\min_{\mathbf{P},\ p_{n\in\Omega}} \sum_{n\in\Omega}\left[f_n(p_n)+\beta_n^0+\sum_{m\in\omega_n}\beta_{nm}p_{nm}\right] \tag{9a}$$

$$\text{s.t.}\quad \mathbf{P}=-\mathbf{P}^\top, \tag{9b}$$

$$p_n=\sum_{m\in\omega_n}p_{nm},\ \forall n\in\Omega, \tag{9c}$$

$$\underline{p_n}\leqslant p_n\leqslant\overline{p_n},\ \forall n\in\Omega, \tag{9d}$$

$$p_{nm}\geqslant 0,\ \forall n\in\Omega_g, \tag{9e}$$

$$p_{nm}\leqslant 0,\ \forall n\in\Omega_c, \tag{9f}$$

$$\left|\mathbf{Gp}\right|\leqslant\bar{l}. \tag{9g}$$

In (9b)–(9f) it coincides with the formulation (2). The primary objectives of algorithm modification are achieved through the introduction of constraint (9g) responsible for line limits, and the regularization of the objective function to introduce agents' preferences and trade-independent fees. The proposed algorithm acts as a unified optimization-based P2P market framework. No corrections and out-of-market interventions are required. The setup works for different types of preferences and can be applied to investigate an influence of energy policies.

4. Distributed P2P market with network constraints, user preferences and network charges

To solve an optimization problem (9a) we design a distributed algorithm based on the consensus alternating direction method of multipliers (ADMM) [8]. Alternative distributed optimization techniques are summarized in [23]. The derivation of the algorithm can be found in Appendix A. The final procedure takes a form

$$\begin{aligned}\mathbf{P}_n^{k+1} &= \operatorname*{argmin}_{\mathbf{P}_n} f_n(p_n)+\beta_n^0+\sum_{m\in\omega_n}[\beta_{nm}p_{nm}+\\ &\lambda_{nm}^k\left(\frac{p_{nm}^k-p_{mn}^k}{2}-p_{nm}\right)+(\rho/2)\left(\frac{p_{nm}^k-p_{mn}^k}{2}-p_{nm}\right)^2\Bigg]+\\ &\frac{\rho_1}{2}t_{1n}+\rho_1\sum_{l\in L}\left(\bar{l}_l-y_{1l}^k+\mu_{1l}^k\right)\left(-\mathbf{G}_{l,n}p_n\right)+\\ &\frac{\rho_1}{2}t_{2n}+\rho_1\sum_{l\in L}\left(\bar{l}_l-y_{2l}^k+\mu_{2l}^k\right)\left(\mathbf{G}_{l,n}p_n\right)\end{aligned} \tag{10a}$$

$$\text{s.t.}\quad p_n=\sum_{m\in\omega_n}p_{nm},\ \forall n\in\Omega \tag{10b}$$

$$\underline{p_n}\leqslant p_n\leqslant\overline{p_n},\ \forall n\in\Omega \tag{10c}$$

$$p_{nm}\geqslant 0,\ \forall n\in\Omega_g \tag{10d}$$

$$p_{nm}\leqslant 0,\ \forall n\in\Omega_c \tag{10e}$$

$$\lambda_{nm}^{k+1}=\lambda_{nm}^k-\rho\left({p_{nm}}^{k+1}+p_{mn}^{k+1}\right)/2 \tag{10f}$$

$$\mathbf{y}_1^{k+1}=\max\left(0,-\mathbf{Gp}^{k+1}+\bar{\boldsymbol{l}}+\boldsymbol{\mu}_1^k\right) \tag{10g}$$

$$\mathbf{y}_2^{k+1}=\max\left(0,\mathbf{Gp}^{k+1}+\bar{\boldsymbol{l}}+\boldsymbol{\mu}_2^k\right) \tag{10h}$$

$$\boldsymbol{\mu}_1^{k+1}=\boldsymbol{\mu}_1^k+\left(-\mathbf{Gp}^{k+1}+\bar{\boldsymbol{l}}-\mathbf{y}_1^{k+1}\right) \tag{10i}$$

$$\boldsymbol{\mu}_2^{k+1}=\boldsymbol{\mu}_2^k+\left(\mathbf{Gp}^{k+1}+\bar{\boldsymbol{l}}-\mathbf{y}_2^{k+1}\right), \tag{10j}$$

where $\mathbf{G}=\mathbf{AI}^\top$, $\mathbf{G}\in\mathbb{R}^{N_l\times N_\Omega}$, $\rho>0$, and expressions for t_{1n} and t_{2n} take a form

$$t_{1n}=\sum_{l\in L}(-\mathbf{G}_{l,n}p_n)^2+2\sum_{l\in L}\sum_{j=n+1}^{\Omega}(-\mathbf{G}_{l,n}p_n)(-\mathbf{G}_{l,j}p_j^k),n\in\Omega\cup n\neq N_\Omega, \tag{11}$$

$$t_{1n}=\sum_{l\in L}(-\mathbf{G}_{l,n}p_n)^2,n=N_\Omega, \tag{12}$$

and

$$t_{2n}=\sum_{l\in L}(\mathbf{G}_{l,n}p_n)^2+2\sum_{l\in L}\sum_{j=n+1}^{\Omega}(\mathbf{G}_{l,n}p_n)(\mathbf{G}_{l,j}p_j^k),n\in\Omega\quad\cup n\neq N_\Omega, \tag{13}$$

$$t_{2n}=\sum_{l\in L}\left(\mathbf{G}_{l,n}p_n\right)^2,\ n=N_\Omega. \tag{14}$$

Expressions (11)–(12), and (13-)-(14) describe one of the possible approaches to distribute $(\sum_{n\in\Omega}-\mathbf{G}_{l,n}p_n)^2$ and $(\sum_{n\in\Omega}\mathbf{G}_{l,n}p_n)^2$ between the agents. Local optimization results

$$\mathbf{P}_n=\begin{pmatrix}p_{n1} & \dots & p_{nN_\Omega}\end{pmatrix}$$

comprise the trades of agent n. Constraints (10b)–(10e) are included in the local optimization problem and calculated at each iteration step. The augmented Lagrangian in (10a) contains terms responsible for reaching the traded power consensus between the agents and terms accounting for an impact of the trades on the network.

We assume utilization of the quadratic cost function. The convergence of the algorithm (10) is ensured for the closed, proper, and convex functions [8]. Global stopping criteria is related to reciprocity requirement (9b)

$$r_n^{k+1}=\sum_{m\in\omega_n}\left(p_{nm}^{k+1}+p_{mn}^{k+1}\right)^2,\quad \sum_{n\in\Omega}r_n^{k+1}\leqslant\varepsilon_r^2, \tag{15}$$

dual residuals

$$s_n^{k+1}=\sum_{m\in\omega_n}\left(p_{nm}^{k+1}-p_{nm}^k\right)^2,\quad \sum_{n\in\Omega}s_n^{k+1}\leqslant\varepsilon_s^2, \tag{16}$$

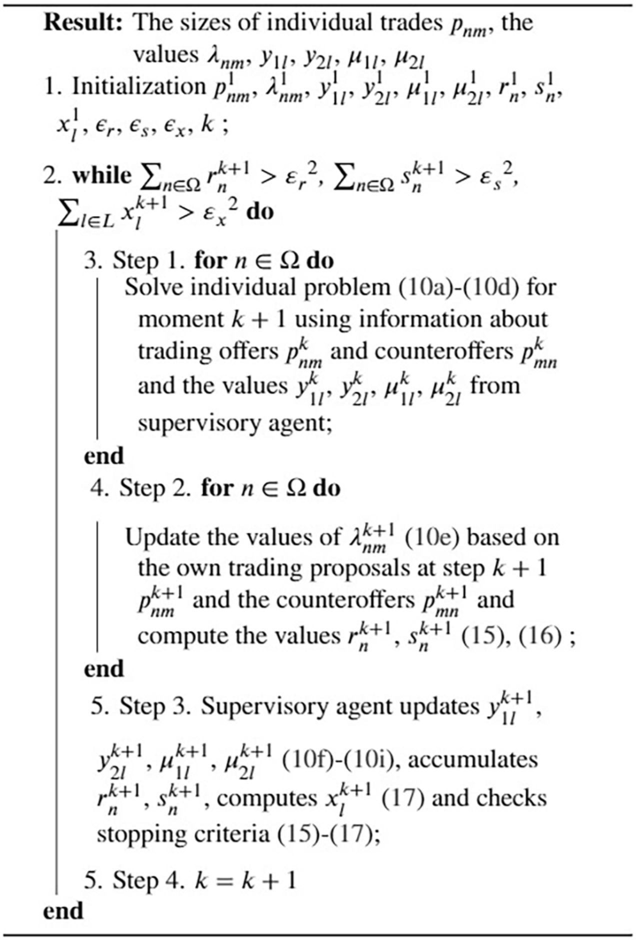
and the values of Lagrangian multipliers $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$

$$x_l^{k+1}=\left(\mu_{1l}^{k+1}-\mu_{1l}^k\right)^2+\left(\mu_{2l}^{k+1}-\mu_{2l}^k\right)^2,\quad \sum_{l\in L}x_l^{k+1}\leqslant\varepsilon_x^2. \tag{17}$$

Each agent, participating in the bilateral market, firstly solves its local optimization problem. Updated values p_{nm}^{k+1}, $m\in\omega_n$ are reported as the trading proposals to the agents from the trading partnership set of agent n. Following (10e), the market participant calculates the values λ_{nm}^{k+1} based on its trading proposal and counteroffer, and computes the residuals (15) and (16), then broadcasted to the trading community. The supervisory agent collects the total trading proposals $\mathbf{p}^{k+1}$ and updates auxiliary variables $\mathbf{y}_1^{k+1}$ and $\mathbf{y}_2^{k+1}$, and Lagrangian multipliers $\boldsymbol{\mu}_1^{k+1}$ and

μ_2^{k+1}. It tests the stopping criteria (17). The simultaneous fulfillment of the criteria (15), (16), (17) indicates that the algorithm has converged to the equilibrium. The flow of the market clearing process can be followed in Algorithm 1.

Algorithm 1. Consensus ADMM algorithm for P2P market with network constraints, user preferences and network charges.



```
Result: The sizes of individual trades $p_{nm}$, the
        values $\lambda_{nm}$, $y_{1l}$, $y_{2l}$, $\mu_{1l}$, $\mu_{2l}$
1. Initialization $p^1_{nm}$, $\lambda^1_{nm}$, $y^1_{1l}$, $y^1_{2l}$, $\mu^1_{1l}$, $\mu^1_{2l}$, $r^1_n$, $s^1_n$,
   $x^1_l$, $\epsilon_r$, $\epsilon_s$, $\epsilon_x$, $k$ ;
2. while $\sum_{n\in\Omega} r_n^{k+1} > \varepsilon_r{}^2$, $\sum_{n\in\Omega} s_n^{k+1} > \varepsilon_s{}^2$,
   $\sum_{l\in L} x_l^{k+1} > \varepsilon_x{}^2$ do
    3. Step 1. for $n \in \Omega$ do
        Solve individual problem (10a)-(10d) for
          moment $k+1$ using information about
          trading offers $p^k_{nm}$ and counteroffers $p^k_{mn}$
          and the values $y^k_{1l}$, $y^k_{2l}$, $\mu^k_{1l}$, $\mu^k_{2l}$ from
          supervisory agent;
    end
    4. Step 2. for $n \in \Omega$ do
        Update the values of $\lambda^{k+1}_{nm}$ (10e) based on
          the own trading proposals at step $k+1$
          $p^{k+1}_{nm}$ and the counteroffers $p^{k+1}_{mn}$ and
          compute the values $r^{k+1}_n$, $s^{k+1}_n$ (15), (16) ;
    end
    5. Step 3. Supervisory agent updates $y^{k+1}_{1l}$,
      $y^{k+1}_{2l}$, $\mu^{k+1}_{1l}$, $\mu^{k+1}_{2l}$ (10f)-(10i), accumulates
      $r^{k+1}_n$, $s^{k+1}_n$, computes $x^{k+1}_l$ (17) and checks
      stopping criteria (15)-(17);
    5. Step 4. $k = k+1$
end
```

5. Test case and application results

This section investigates the behavior of the proposed P2P market algorithm. We examine how the map of trades is affected by user preferences and network charges, investigate the behavior of the algorithm with the presence of congestions in the network and compare it with the correction algorithm.

5.1. Test case

To run the market, we use the IEEE 39-bus power system with flexible generation and loads. The community of agents Ω contains 10 generators and 21 loads. All agents in the test case have quadratic cost function, where parameters a_n, b_n, and operational limits are summarized in Table 3 in Appendix B, p_n represents a total amount of power traded by agent n

$$f_n(p_n) = \frac{1}{2}a_n p_n^2 + b_n p_n. \tag{18}$$

The total trades of generators and loads lay in their power boundaries. The test system contains 46 lines with parameters specified in [20]. We set the line limits to be high enough for the lines not to be overloaded, except the limit for the specific line described in the simulation cases further. We run the simulation setup in MATLAB on personal computer (Intel Core i5-7200U CPU, 2.50 GHz, RAM 8 GB, 64-bit Operating System). Not considering parallelization (individual agents' tasks are solved sequentially on one computer), algorithm (10) converges with the primal and dual tolerances 10^{-5} and power flow tolerance 10^{-3} ($\rho = 1.5$, $\rho_1 = 0.00028$) in 59 s. In this way, considering parallelization, it takes around 2 s for the market to be cleared, which is adequate for the P2P market and has potential for scaling. Optimization of individual problems was done using built-in MATLAB function *quadprog*.

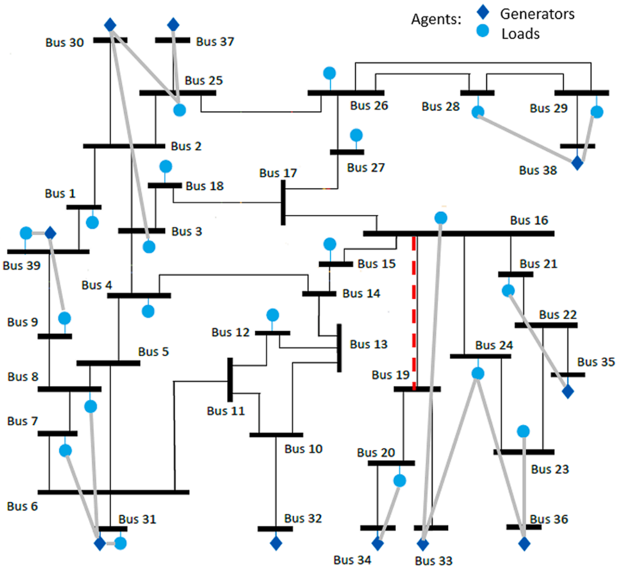


Fig. 2. New England test case. Map of P2P trades without network constraints, with equal user preferences and network charges.

5.2. P2P market without network constraints, user preferences and network charges

The first test case describes the P2P market without congestion management, trading preferences, and network charges. The map of

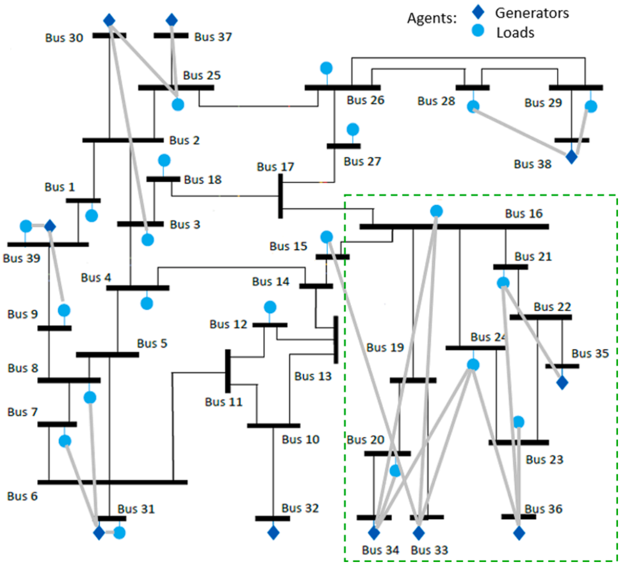


Fig. 3. New England test case. Map of P2P trades without network constraints, with network charges and different user preferences.

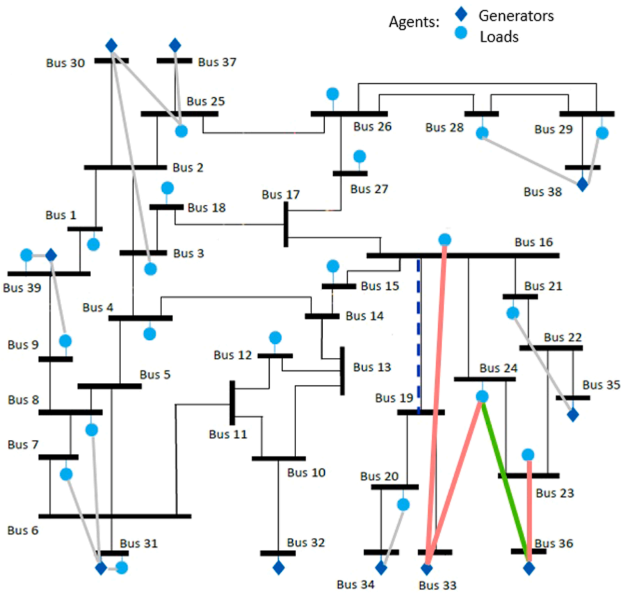


Fig. 4. New England test case. Map of P2P trades with active network constraints.

trades, in this case, is quite dense and not presented here. The most explicit description is that each generating agent trades with each consumer from its partnership set ω_n. The market converges to a uniform price.

5.3. P2P market without network constraints, with user preferences and network charges

We propose a framework to account for line overloads, user preferences, and trade-independent fees in the one-step algorithm. To assess an impact of algorithm extensions, we apply them separately. In this subsection, we examine how the matrix of trades is affected by user preferences and network charges. We apply the distance-based user preferences for all agents in the system and set the values of the distance unit fees equal to $s_{nm} = 10$. Trade-independent values of network charges are chosen to be $\beta_n^0 = 0.1, n \in \Omega$. This approach results in the set of power trades presented in Fig. 2. Comparison with the market without preferences and network fees from Section 5.2 states that the number of trades significantly decreases. For the chosen parameters of distance unit fees, agents prefer to trade with a low number of neighbors forming small clusters of trades. The exogenously enforced agents' preferences make the trading with agents out of preferences more expensive and, therefore, disadvantageous, suggesting its redistribution or refusal. More generally, in the market with flexible generation and demand, agents' preferences lead to the decrease of total power traded in the market. Setting the line limit 16–19 to 200 MW one can observe an overload.

Fig. 3 illustrates the changes in trades when the distance unit fee for the agents within the zone specified by green dashed box is decreased to $s_{nm} = 5$. In comparison with Fig. 2, the number and the value of some trades in this zone increases. We observe the new trades between agents at buses 33 –15, 34 –16, 34 –24, 36 –21, as well as an increase of trade 35 –21. The values of distance unit fees s_{nm} determine the clustering of the agents and intensity of trades. The proposed regularization-based approach works effectively ensuring both individual and collective user preferences.

5.4. P2P market with network constraints, user preferences and network charges

As was stated early, the application of algorithm (9a) or (10) without network constraints for the case with equal distance unit fees leads to an overload of the line 16–19. With activated congestion management we can observe the new map of power trades, shown in Fig. 4. The overall pattern of trades is similar to the case in Fig. 2. The congestion is eliminated, resulting in a slight decrease in total welfare from 27979.6 with overloaded line 16-19 to 27955 with active congestion management. To address the overload, the trades between the agents at buses 33 and 16, and 33 and 24 were decreased. Since the load at bus 24 is ready to pay a higher price than the load at bus 23, in a situation when the trade between agents at buses 33 and 24 was decreased due to the congestion management requirements, the agent at bus 36 uses an opportunity to increase trade with load at bus 24, in this way, the congestion management framework matches the logic of the market. In Fig. 4, the decreased trades are shown in solid red lines while the increased trades - in green lines. The dashed blue line denotes the cleared congestion. The proposed approach works for several active network constraints.

Algorithms (9a) and (10) represent the P2P electricity market framework with built-in congestion management, user preferences, and trade-independent fees. This approach results in the feasible trades, do not require correction actions and works in a distributed manner. To assess the proposed algorithm, we can compare it with an algorithm from another branch of approaches - algorithm of post-trade correction.

5.5. Correction alternative

The decentralized procedure (10) defines a P2P market with network constraints, user preferences, and trade-independent fees. It ensures the simultaneous fulfillment of economic and operational decisions, proposing a one-step solution. One can notice that the same logic of using a matrix of loading vectors can be applied to correct initial market proposals if the market architecture does not contain network-related constraints, in our case (9g). Firstly the logic of correction was proposed in [34] to modify power flows after the free trades. It is considered as a method to achieve a division between the economic and feasibility decisions. In this case, an optimization algorithm without network constraints will be supplemented by the second-step optimization problem. The advantages of the approach, besides simplification of the algorithm and objectives separation, include lowering of communication overheat.

At the first stage, the declared trades and the sizes of congestion through the lines are determined as a result of power flow calculation. The second stage executes the curtailment of the trades following the chosen logic as minimum correction cost, the minimum size of correction, and others. The second-stage algorithm which minimizes the cost of correction takes a form

$$\min_{\Delta p_n} \sum_{n\in\Omega}\left[f_n\left(p_n+\Delta p_n\right)+\sum_{m\in\omega_n}\beta_{nm}\left(p_{nm}+\Delta p_{nm}\right)\right. \\ \left. -f_n\left(p_n\right)-\sum_{m\in\omega_n}\beta_{nm}p_{nm}\right] \tag{19a}$$

$$\text{s.t.} \quad \mathbf{\Delta P} = -\mathbf{\Delta P}^{\top}, \tag{19b}$$

$$\Delta p_n = \sum_{m\in\omega_n}\Delta p_{nm}, \forall n \in \Omega, \tag{19c}$$

$$\underline{p_n} \leqslant p_n + \Delta p_n \leqslant \overline{p_n}, \forall n \in \Omega, \tag{19d}$$

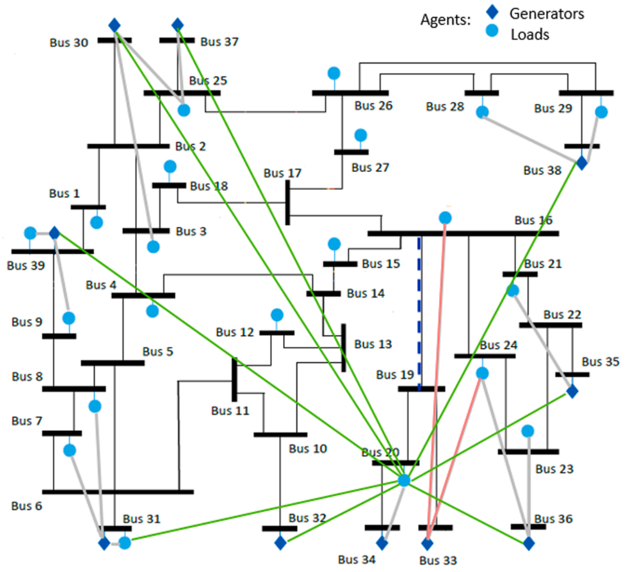


Fig. 5. Map of P2P trades with active network constraints under the minimum size of correction algorithm.

Table 2
Comparison of congestion management algorithms.

Parameter	Built-in congestion management	Correction-based approach
Total welfare, $	27955	26481
Cost of correction, $	24.6	1498.6
Number of trades, participated in correction	4	10
Average size of correction trade, MW	9.87	2.24

$$p_{nm} + \Delta p_{nm} \geqslant 0, \forall n \in \Omega_g, \tag{19e}$$

$$p_{nm} + \Delta p_{nm} \leqslant 0, \forall n \in \Omega_c, \tag{19f}$$

$$\left|\mathbf{G}\left(\Delta\mathbf{p} + \mathbf{p}\right)\right| \leqslant \bar{l}, \tag{19g}$$

where Δp_n is a correction of the total amount of power traded by agent n, Δp_{nm} depicts the correction of individual trades with agents m within the trading partnership set ω_n. Elements (n,m) of matrix $\Delta\mathbf{P}$ correspond to the values Δp_{nm}, $\bar{l}$ denotes line flow limits.

Frequently in the P2P electricity market, agents do not want to disclose their cost functions to curtailment agent (DSO) to maintain privacy and exclude the possibility for gaming. In this case, the logic of the minimum size of correction is usually proposed

$$\min_{\Delta p_n} \quad \sum_{n\in\Omega}\sum_{m\in\omega_n}\left|\Delta p_{nm}\right| \tag{20a}$$

$$\text{s.t.} \quad \Delta\mathbf{P} = -\Delta\mathbf{P}^{\top} \tag{20b}$$

$$\Delta p_n = \sum_{m\in\omega_n}\Delta p_{nm}, \ \forall n \in \Omega \tag{20c}$$

$$\underline{p_n} \leqslant p_n + \Delta p_n \leqslant \overline{p_n}, \ \forall n \in \Omega \tag{20d}$$

$$p_{nm} + \Delta p_{nm} \geqslant 0, \ \forall n \in \Omega_g \tag{20e}$$

$$p_{nm} + \Delta p_{nm} \leqslant 0, \ \forall n \in \Omega_c \tag{20f}$$

$$|\mathbf{G}(\mathbf{p} + \Delta\mathbf{p})| \leqslant \bar{l}. \tag{20g}$$

The minimum correction cost algorithm (19), after the cost-based decisions at the first stage, results in the same map of power trades as depicted in Fig. 4. The trading pattern for the minimum size of correction approach is shown in Fig. 5. Algorithm (20a) also eliminates congestion, however, the map of trades differs from Fig. 4. Similarly to the previous case, there is a reduction in the power trades between the agents at buses 33 and 24, and 33 and 16. However, the sizes of these corrections are smaller than in the preceding case, which is shown by the thickness of red lines. Additionally, we observe new trades initiated to reduce the congestion of the line 16–19. The sizes of these trades are small (thin green lines in Fig. 5).

Table 2 summarizes the results of congestion management performed by the proposed one-step algorithm and correction algorithm (20a). We can see that algorithm with built-in congestion management achieves the minimum cost of correction, preserving as much as possible the total welfare. The algorithm (20a) is aimed to adjust initial trades to satisfy network constraints with minimal intervention. This problem setup, as shown in Table 2, can lead to suboptimality, increasing the cost of congestion management, and the number of trades involved in the correction. At the same time, the algorithm achieves its primary goal of correction with minimum intervention, resulting in the decreased average size of correction trade.

Although additional correction conditions can be specified in the formulation (20a) as a limited number of trades participating in correction and others, without the utilization of information about agents' cost functions, there is a risk of an expensive correction. Thus, however, a variety of not cost-based approaches can be offered for the correction of P2P trades one needs to investigate their possible influence on the trading pattern and total welfare. Besides that, correction-based P2P market requires the calculation of power flow distribution after the initial trades to reveal the possible overload, which lengthens the process of trading. In contrast to correction-based approaches the proposed market design avoids intermediate power flow calculation.

The suggested one-stage algorithm provides a solution which coincides with the solution of algorithm (19) and surpasses in terms of total welfare the solution in the case of minimum size of correction logic (20a).

6. Discussion

The paper aims to design a P2P market architecture able to work with the presence of network constraints. For this purpose, we propose a distributed framework, when each agent firstly solves its local problem. Agents exchange their trading proposals p^k_{nm} across the trading partnership set ω_n and utilize the values of total trading proposals p^k_j, $j \in \Omega$, $j \neq n$. To solve the local optimization problem agents do not need information about the division of the total trade p^k_j between the individual trades and the cost functions of other agents which make information exchange applicable for the market applications.

At the same time during this iterative process the leakage of data privacy still may arise. To address this problem privacy preserving modifications of the algorithm could be proposed. Authors in [12] propose an algorithm where each agent approximately solves a perturbed optimization problem that is formulated from its local private data in an iteration, and then perturbs the approximate solution with

Gaussian noise to provide the distributed privacy guarantee. In [35] the methods of dual variable perturbation and primal variable perturbation to provide dynamic differential privacy are proposed. If we have incomplete data due to privacy issues or communication problems, we can apply the heuristic solutions. We can use a trading proposal from the previous iteration step or calculate projection of the proposal. Several consecutive data skips may lead to the agent's exclusion from the trading set for this market calculation round. Alternatively, the Bayesian games approach could be applied for the P2P market.

An optimization problem (9), (10) is a general formulation of the P2P market with network constraints, user preferences and network charges. Besides the consensus ADMM approach it can be distributed using other methods described in [23]. Several extensions and convergence rate improvements can be proposed following [15,3]. The interval of calculation can be reduced by:

- Introducing either y_{1l}, μ_{1l} or y_{2l}, μ_{2l} for all or several lines in the system;
- Applying congestion management for the critical lines or lines with a high probability of overload;
- Using warm starts based on the historical data;
- Reducing the trading partnership set ω_n of agent n to the "most probable" graph of trades based on the historical information;
- Varying penalty parameters ρ and ρ_1 independently or jointly to ensure the fastest convergence [7].
- Applying over-relaxation [7].
- Applying accelerated versions of ADMM algorithm [19], that demonstrated faster convergence for distributed state estimation problem in power systems [30]

The proposed P2P market could be adopted for distribution systems following one of the methods [22,5,14]. The PTDF approach to control line overloads could be applied in low-voltage networks. The scheme can be extended to account for voltage through the voltage sensitivity matrix and account for losses through the loss sensitivity matrix. All market mechanisms proposed in work will remain functional after the changes.

In this article we assume that the lines are lossless. If we relax this assumption one can apply the loss allocation approaches proposed in [34,16]. In the current formulation, similarly to the trade-independent network fees, there could be proposed trade-independent slack-bus payments aiming to compensate losses; however, this approach can not be considered as the fair allocation of losses. Additionally, other types of user preferences can be introduced and investigated.

7. Conclusion

With an increase of distributed generation, growing attention is paid to the possibilities of its utilization in the network. The P2P electricity market represents one of the possible ways to address this question. This work focuses on the design of the P2P electricity market, offering more independence and freedom of action to market participants. The P2P trading scheme enables new types of services and proposes additional value as differentiated contracts, enforced consumer preferences, and increased utilization of distributed generation.

In this paper, we propose a P2P market design, incorporating network constraints, user preferences, and trade-independent fees. In this way, we ensure a meeting of three requirements critical to the practical implementation of the P2P markets as secure operation, consumer-centric nature of the market, and the provision of benefits for the grid. We propose a distributed framework and compare the results with an alternative correction-based algorithm. The simulation results demonstrate the successful elimination of line congestions and show the advantage of the one-step algorithm with built-in congestion management. We propose an effective way to ensure individual and collective agents' preferences and include trade-independent fees. The algorithm could be adopted for the unbalanced distribution networks and extended for other types of user preferences.

CRediT authorship contribution statement

Tatiana Chernova: Conceptualization, Investigation, Software, Writing - review & editing. **Elena Gryazina:** Conceptualization, Methodology, Supervision, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

We propose to use a consensus ADMM approach to address the problem (9a) and develop decentralized procedure. While a method to incorporate (9b) in the augmented Lagrangian and distribute it between the agents discussed in [6], the steps required to include the network constraints into the framework can be found below.

Using notation $\mathbf{G} = \mathbf{A}\mathbf{I}^{\top}$, where $\mathbf{A}$ is the matrix of loading vectors, $\mathbf{I}$ describes the correspondence between buses and agents, line power flow constraints read

$$\left|\mathbf{G}\mathbf{p}\right| \leqslant \bar{\boldsymbol{l}}. \tag{21}$$

Then moving from the absolute values of power flows, introducing slack variables for the inequalities, expression (21) comes to the form

$$\begin{cases} -\mathbf{G}\mathbf{p} + \bar{\boldsymbol{l}} - \mathbf{y}_1 = 0 | \boldsymbol{\mu}_1, \ \mathbf{y}_1 \geqslant 0 \\ \mathbf{G}\mathbf{p} + \bar{\boldsymbol{l}} - \mathbf{y}_2 = 0 | \boldsymbol{\mu}_2, \ \mathbf{y}_2 \geqslant 0. \end{cases} \tag{22}$$

Based on (22) the Lagrangian will be augmented by

$$\mathbf{L_{add}} = \frac{\rho_1}{2}\left|\left| -\mathbf{G}\mathbf{p} + \bar{\boldsymbol{l}} - \mathbf{y}_1^k + \boldsymbol{\mu}_1^k \right|\right|^2 + \frac{\rho_1}{2}\left|\left| \mathbf{G}\mathbf{p} + \bar{\boldsymbol{l}} - \mathbf{y}_2^k + \boldsymbol{\mu}_2^k \right|\right|^2, \tag{23}$$

with the addition of extra equations to the algorithm

$$\mathbf{y}_1^{k+1} = \max\left(0, -\mathbf{G}\mathbf{p}^{k+1} + \bar{\boldsymbol{l}} + \boldsymbol{\mu}_1^k\right), \tag{24}$$

$$\mathbf{y}_2^{k+1} = \max\left(0, \mathbf{G}\mathbf{p}^{k+1} + \bar{\boldsymbol{l}} + \boldsymbol{\mu}_2^k\right), \tag{25}$$

$$\boldsymbol{\mu}_1^{k+1} = \boldsymbol{\mu}_1^k + \left(-\mathbf{G}\mathbf{p}^{k+1} + \bar{\boldsymbol{l}} - \mathbf{y}_1^{k+1}\right), \tag{26}$$

$$\boldsymbol{\mu}_2^{k+1} = \boldsymbol{\mu}_2^k + \left(\mathbf{G}\mathbf{p}^{k+1} + \bar{\boldsymbol{l}} - \mathbf{y}_2^{k+1}\right). \tag{27}$$

Continuing derivations for the first term of (23), one can obtain

$$\frac{\rho_1}{2}\left\| -\mathbf{G}\mathbf{p} + \bar{\boldsymbol{l}} - \mathbf{y}_1^k + \boldsymbol{\mu}_1^k\right\|^2 = \frac{\rho_1}{2}\sum_{l\in L}\left(\sum_{n\in\Omega} -\mathbf{G}_{l,n}p_n + \bar{l}_l - y_{1l}^k + \mu_{1l}^k\right)^2. \tag{28}$$

Expression (28) can be written as

$$\frac{\rho_1}{2}\sum_{l\in L}\left(\sum_{n\in\Omega} -\mathbf{G}_{l,n}p_n + \bar{l}_l - y_{1l}^k + \mu_{1l}^k\right)^2 = \frac{\rho_1}{2}\sum_{l\in L}(\alpha_l + \eta_l)^2 = \frac{\rho_1}{2}\sum_{l\in L}\left(\alpha_l{}^2 + 2\alpha_l\eta_l + \eta_l^2\right), \tag{29}$$

where $\alpha_l = \sum_{n\in\Omega} -\mathbf{G}_{l,n}p_n$, $\eta_l = \bar{l}_l - y_{1l}^k + \mu_{1l}^k$. In the next step

$$\begin{aligned}\frac{\rho_1}{2}\sum_{l\in L}(\alpha_l{}^2 + 2\alpha_l\eta_l + \eta_l^2) &= \frac{\rho_1}{2}\sum_{l\in L}\left(\left(\sum_{n\in\Omega} -\mathbf{G}_{l,n}p_n\right)^2 + 2\left(\sum_{n\in\Omega} -\mathbf{G}_{l,n}p_n\right)\eta_l + \eta_l^2\right)\\ &= \frac{\rho_1}{2}\left(\sum_{l\in L}\left(\sum_{n\in\Omega} -\mathbf{G}_{l,n}p_n\right)^2 + 2\sum_{n\in\Omega}\sum_{l\in L}\eta_l\left(-\mathbf{G}_{l,n}\right)p_n + \sum_{l\in L}\eta_l^2\right).\end{aligned} \tag{30}$$

The first term of (30) after the distribution between the agents takes a form for $n \neq N_\Omega$

$$t_{1n} = \sum_{l\in L}\left(-\mathbf{G}_{l,n}p_n\right)^2 + 2\sum_{l\in L}\sum_{j=n+1}^{\Omega}\left(-\mathbf{G}_{l,n}p_n\right)\left(-\mathbf{G}_{l,j}p_j^k\right), \tag{31}$$

if $n = N_\Omega$, then

$$t_{1n} = \sum_{l\in L}\left(-\mathbf{G}_{l,n}p_n\right)^2. \tag{32}$$

We can repeat the same procedure for the second term of (23) getting

$$t_{2n} = \sum_{l\in L}\left(\mathbf{G}_{l,n}p_n\right)^2 + 2\sum_{l\in L}\sum_{j=n+1}^{\Omega}\left(\mathbf{G}_{l,n}p_n\right)\left(\mathbf{G}_{l,j}p_j^k\right), \quad n \neq N_\Omega, \tag{33}$$

and

$$t_{2n} = \sum_{l\in L}\left(\mathbf{G}_{l,n}p_n\right)^2, n = N_\Omega. \tag{34}$$

Applying (31) and (32), (33) and (34), and omitting $\sum_{l\in L}\eta_l^2$, the decentralized P2P market algorithm takes a form (10).

Appendix B

Parameters of the P2P market agents are summarized in Table 3. It contains information about the agents' location, parameters of their cost functions, and operational limits.

Table 3
Parameters of the agents [6].

Agent	Bus	a_n $[\$/\mathrm{MW}^2]$	b_n $[\$/\mathrm{MW}]$	$\underline{P_n}$[MW]	$\overline{P_n}$[MW]
1	1	0.067	64	−146.4	0
2	3	0.047	79	−483	0
3	4	0.047	71	−750	0
4	7	0.053	62	−350.7	0
5	8	0.082	65	−783	0
6	9	0.052	83	−9.8	0
7	12	0.087	63	−12.8	0
8	15	0.057	81	−480	0
9	16	0.050	73	−493.5	0
10	18	0.052	69	−237	0
11	20	0.071	62	−1020	0
12	21	0.064	79	−411	0
13	23	0.057	60	−371.3	0
14	24	0.082	80	−462.9	0
15	25	0.069	78	−336	0
16	26	0.069	70	−208.5	0
17	27	0.086	62	−421.5	0
18	28	0.054	70	−309	0
19	29	0.078	66	−425.3	0
20	31	0.081	70	−13.8	0
21	39	0.059	71	−1656	0
22	30	0.089	18	0	1040
23	31	0.067	21	0	646
24	32	0.055	37	0	725
25	33	0.082	25	0	652
26	34	0.088	17	0	508
27	35	0.076	38	0	687
28	36	0.084	28	0	580
29	37	0.077	36	0	564
30	38	0.051	38	0	865
31	39	0.087	19	0	1100

Appendix C. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.ijepes.2021.106981.

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